Minimax Feature Merge: The Featural Linguistic Turing Machine

Louis Van Steene, University of Cambridge
lecv2@cam.ac.uk / louis-vs.github.io
ULAB – 16th April 2021

You can download this handout by following the link above. Feel free to ask any questions in the ULAB Discord or by emailing.

1. Background

The ultimate goal: formalising the primitive symbols and procedures of linguistic computation, by taking an interdisciplinary approach to understanding the nature of language and its place in the mind/brain.

What is an I-language? (internal, intensional, individual)

Chomsky (2000; 2001):

L = a device that generates a set of expressions

L is a state of the faculty of language ("a component of the human mind/brain dedicated to language", Chomsky 2000: 89)

(1) An expression Exp is an ordered pair $\langle PF, LF\rangle$ where PF and LF are legible inputs to the interface between the syntactic and sensorimotor SM or conceptual-intentional C-I modules, respectively. The syntactic module L generates an infinite set of expressions $\{Exp\}$.

(2) The Strong Minimalist Thesis (SMT): Syntax is an optimal solution to legibility conditions imposed by the phonological and semantic modules.

(3) UG ('universal grammar') is the initial state of FL.

Expressions are built with the function $Merge(X, Y) = \{X, Y\}$, where X and Y are 'lexical items' (LIs).

The Lexicon: "Lex [is] in principle 'Bloomfieldian', a 'list of exceptions' that provides just the information required to yield the interface outputs and does so in the best way, with least redundancy and complication" (Chomsky 2001: 10)

But Chomsky supposes another operation which builds LIs, and then assumes that these are the building blocks of syntax (the inputs to Merge).

The approach I take here assumes that these operations are one and the same – in line with Distributed Morphology (Marantz, 1997) and Nanosyntax (Baunaz et al, 2018).

Biolinguistics: Biolinguistics refers to the study of "language as a natural phenomenon—an aspect of [man's] biological nature, to be studied in the same manner as, for instance, his anatomy." (Lenneberg, 1967, p. vii).

'Mathematical biolinguistics' – Jeffrey Watumull.

“[w]hat matters in representations is form, not substance” (Gallistel, 2001, p. 9692).

“*if you can’t program it, you haven’t understood it*” (Deutsch, 2011, p. 146, emphasis original). Importantly, in this context, the ‘program’ must have the quality of *strong generativity*.

2. Architecture

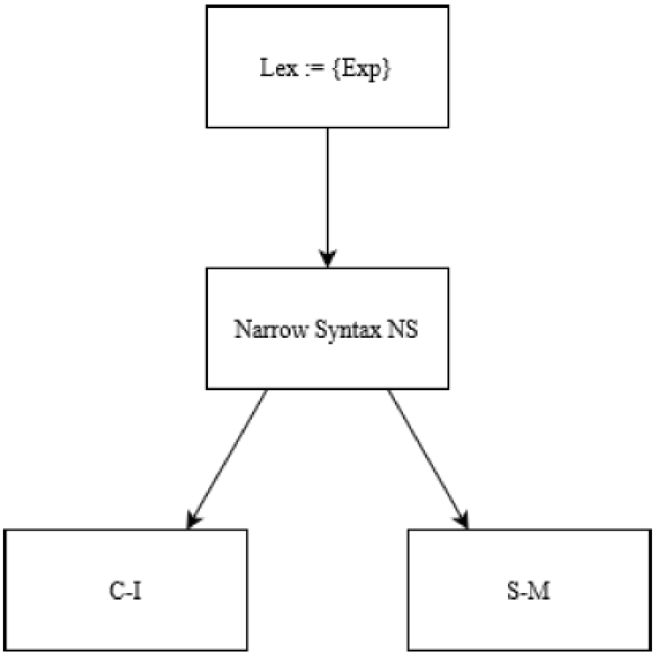

Figure 1: *The Y-model.*

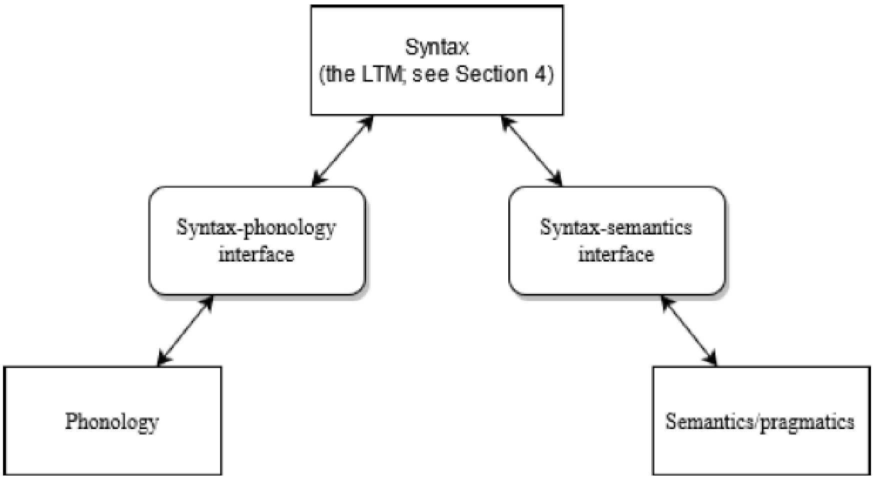

Figure 2: *The architecture.*

A computational module is equivalent to the function f : X -> X; an interface is equivalent to the function g : X -> Y, XnY={}. Different interfaces use different sets of symbols corresponding to X and Y; however, the general form of the representation is *isomorphic* between all instantiations.

3. The LTM and the featural LTM

Watumull's (2015) LTM:

(4) The mathematical hypotheses

(i) The External Reality Hypothesis (ERH)

"*There exists an external physical reality completely independent of us humans.*" (Tegmark, 2014, p. 254, emphasis original)

(ii) The Mathematical Universe Hypothesis (MUH)

"*Our external physical reality is a mathematical structure.*" (Tegmark, 2014, p. 254, emphasis original)

(iii) The Mathematical Language Hypothesis (MLH)

"FL is a mathematical structure" (Watumull, 2015, p. 2).

(5) The linguistic Turing machine LTM is the quintuple $(Q, \Gamma, \delta, \#S, \#H)$

Q: states

Γ: symbols (not including the blank symbol, see Sipser, 2012, p. 168)

δ: transition function, from one state/symbol pair to another

$\#_S$: start symbol

$\#_H$: halt symbol[5]

The LTM encodes the computable function f_{MERGE}, defined as in Example 6:[6]

(6) $f_{MERGE}(X, Y) = X, Y$ for $X, Y \in \Gamma$

My LTM:

(7) F is the set of syntactic primitives such that $|F| \geq 1$.

(8) The members of F comprise the urelements in S.

(9) S is defined as the union of F with the result of the inductive application of f_{MERGE} to F.

(10) $f_{MERGE} : S \times S \rightarrow S$

f_{MERGE} is a function that takes two members of S as input and returns a member of S as output.

$f_{MERGE}(X, Y) = \{X, Y\}$ for $X, Y \in S$

The featural LTM, as with the LTM in Example 5, is isomorphic to a free magma. A magma is the most basic mathematical structure in universal algebra, consisting of a set S and a binary operation closed under S. A magma is labelled *free* if no other restrictions are applied (cf. Watumull, 2015, p. 7).

(11) The syntactic component L is isomorphic to the free magma (S, f_{MERGE}), which in turn is isomorphic to the featural LTM.

5. Mathematical biolinguistics

Computation = the Turing machine

See Gallistel & King (2010) for a computational approach to neuroscience, which heavily inspires this approach. This gives confidence that this kind of approach will lessen the 'granularity mismatch problem' highlighted by Poeppel & Embick (2005).

Under the mathematical hypotheses in (4), "the theory needs to be mathematical because the phenomenon is mathematical" (Watumull, 2012: 229).

Minsky & Bobrow experiment (see Watumull & Chomsky, 2020 and Roberts et al, in press):

The 'useful' areas of computational space consist mostly, perhaps entirely, of functions which reduce to the successor function (and thus to f_{MERGE} as in (8)). From an *evolutionary* perspective, then, the odds that the useful computation chanced upon by random genetic mutation is f_{MERGE} are relatively high. This affords independent support, then, to the SMT, and the devolution of as much as possible to the interfaces, as described in Section 3.

(12) Given a set x, the successor of x, written as x^+, is the set

$$x^+ = x \cup \{x\}$$

So

$$0 = \varnothing$$
$$1 = \varnothing^+ = \varnothing \cup \{\varnothing\} = \{\varnothing\}$$
$$2 = (\varnothing^+)^+ = \{\varnothing\} \cup \{\{\varnothing\}\} = \{\varnothing, \{\varnothing\}\}$$
$$3 = (\varnothing^{++})^+ = \{\varnothing, \{\varnothing\}\} \cup \{\{\varnothing, \{\varnothing\}\}\} = \{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$$
$$\ldots$$

Using f_{MERGE}

(13)
$$0 = \varnothing$$
$$1 = F_1$$
$$2 = f_{MERGE}(F_1, \varnothing) = \{F_1, \varnothing\}$$
$$3 = f_{MERGE}(\{F_1, \varnothing\}, \varnothing) = \{\{F_1, \varnothing\}, \varnothing\}$$

Indeed, this formulation arguably makes maximal use of *more minimal* means, by only requiring a single feature, whereas Watumull's (2015, p. 86) appears to require multiple LIs (LI_1, LI_2, $\ldots$).

Another important consequence – a free magma can be generated even if **F** only has one feature in it. This could serve as a 'bootstrapping' function and implies that any complications to the structure of the featural Turing machine are the result of *external* necessity.

6. Conclusion

- Language is (or at the very least can be considered as) a mathematical object.
- The *form* of the syntactic component is isomorphic to the featural Linguistic Turing Machine, which is isomorphic to f_{MERGE}, which is isomorphic to the free magma (**F**, f_{MERGE}).
- This formulation of Merge is 'minimax' from the perspective of the tension between evolvability and learnability.
- A clear area for improvement is in empirical grounding: what kind of tests can differentiate this theory from others in the generative framework?

References

Adger, D., & Svenonius, P. (2011). Features in minimalist syntax. In C. Boeckx (Ed.), *The Oxford handbook of linguistic minimalism* (pp. 27–51). Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199549368.013.0002

Baunaz, L., Haegeman, L., De Clercq, K., & Lander, E. (Eds.). (2018). *Exploring nanosyntax*. Oxford University Press. https://oxford.universitypressscholarship.com/view/10.1093/oso/9780190876746.001.0001/oso-9780190876746

Boeckx, C. (2014). *Elementary syntactic structures: Prospects of a feature-free syntax*. Cambridge University Press. https://doi.org/10.1017/CBO9781139524391

Chomsky, N. (2000). Minimalist inquiries: The framework. In R. Martin, D. Michaels, & J. Uriagereka (Eds.), *Step by step: Essays on minimalist syntax in honor of Howard Lasnik* (pp. 89–156). MIT Press.

Chomsky, N. (2001). Derivation by phase. In M. J. Kenstowicz (Ed.), *Ken Hale: A life in language* (pp. 1–52). MIT Press.

Chomsky, N. (2005). Three factors in language design. *Linguistic Inquiry*, *36*(1), 1–22. https://doi.org/10.1162/0024389052993655

Deutsch, D. (2011). *The beginning of infinity: Explanations that transform the world*. Allen Lane.

Gallistel, C. R. (2001). Mental representations, Psychology of. In N. J. Smelser & P. B. Baltes (Eds.), *International Encyclopedia of the Social & Behavioral Sciences* (pp. 9691–9695). Pergamon. https://doi.org/10.1016/B0-08-043076-7/01488-1

Gallistel, Charles R., & King, A. P. (2010). *Memory and the computational brain: Why cognitive science will transform neuroscience*. Wiley-Blackwell.

Marantz, A. (1997). No escape from syntax: Don't try morphological analysis in the privacy of your own lexicon. *University of Pennsylvania Working Papers in Linguistics*, *4*(2), 14.

Poeppel, D., & Embick, D. (2005). Defining the relation between linguistics and neuroscience. In A. Cutler (Ed.), *Twenty-First century psycholinguistics: Four cornerstones* (pp. 103–118). Taylor & Francis Group. http://ebookcentral.proquest.com/lib/cam/detail.action?docID=3060658

Roberts, I., Watumull, J., & Chomsky, N. (in press). Universal Grammar. In D. Vakoch (Ed.), *Xenolinguistics: Toward a science of extraterrestrial language*. Oxford University Press.

Scheer, T. (2020). On the lexical character of intermodular communication. *Radical: A Journal of Phonology*, *1*, 183–239.

Watumull, J. (2012). A Turing program for linguistic theory. *Biolinguistics*, *6*(2), 222–245.

Watumull, J. (2013). Biolinguistics and Platonism: Contradictory or consilient? *Biolinguistics*, *7*, 301–315.

Watumull, J. (2015). *The linguistic Turing Machine* [PhD Dissertation]. University of Cambridge.

Watumull, J., & Chomsky, N. (2020). Rethinking universality. In A. Bárány, T. Biberauer, J. Douglas, & S. Vikner (Eds.), *Syntactic architecture and its consequences II: between syntax and morphology* (pp. 3–24). Language Science Press. https://langsci-press.org/catalog/book/276